

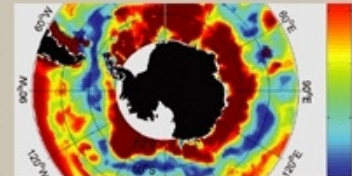
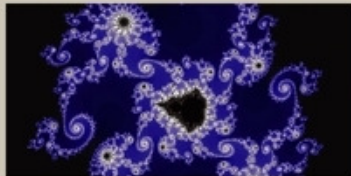
# Discussion of presentations by Jeremy Oakley and Brian Reich

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# Finding the important variables

Both papers seek to identify what variables are important, primarily in a computer model:

- Reich, Storlie & Bondell (RSB) seek variable selection in a more traditional sense (largest impact on predictions)
- Oakley seeks variables that *matter*, in terms of utility, loss and decisions.

My experience:

Increased model flexibility makes variable selection harder.

- Balancing act that both papers seem to have accomplished.
- Many modern methods have ad-hoc (but effective) indices of variable importance, mostly via predictive accuracy.

# "Linear" components to the models

Both papers consider

$$\text{Response} = \text{linear model} + \text{GP error}$$

Linear model =  $\beta_1 h_1(x) + \beta_2 h_2(x) + \dots + \beta_p h_p(x)$ , for known  $h$ 's

- RSB consider linear (main effect), quadratic, and cross-product (2fi) terms.
- Oakley considers linear (main effect) terms only.

Remainder term of the model is a GP that absorbs nonlinearity and higher-order interactions.

Potential problems:

- Explosion of cross-products as number of predictors grows.

# Getting the model right

What if the true model isn't close to the kind of model being fit?

- Oakley's approach uses the model to measure practical importance of each variable.
  - What if the model's not right?
- Linear + GP is a reasonably flexible class, but may miss:
  - Additive nonlinear functions with large  $p$ .
  - Sum of functions with multiple modes.
- ... but see next slide...

# Validation

- Both papers checked their models:
  - RSB via simulation
  - Oakley by computing partial EVPI and EVSI using the true computer model.
- I liked this approach.
- In-sample methods will be problematic
  - Assessing accuracy of an interpolating approximation isn't easy.
- Would conventional validation strategies (cross-validation) accomplish this when the above approaches are too expensive?

# Scaling in $n$ and $p$

GPs :

- computationally expensive as  $n$  increases
- GP predictions are based on measuring distance in  $p$  dimensions.
  - For big  $p$ , distance-based methods (nearest neighbours) can predict poorly.
  - Models that assume additivity may do better (a strong point of both methods).
- How would the models do if we had  $p=50$  predictors?

# Error distributions

In RSB, error was either zero (application) or nonzero (simulations). Does this change the interpretation of the model?

- Or can you get away with saying that your residual error is zero and using a flexible model?

In Oakley, are the covariates assumed to be independent?

# Performance comparison – BART

(test set MSE;

for comparison MSE of constant model is about 16)

	BSS-ANOVA	Linkletter	BART	BART $p=50$
1 / iid	1.67	3.50	2.61	4.52
1 / equicorr	4.11	7.39	4.48	6.75
1 / AR	3.72	6.38	4.30	7.29
2 / iid	1.63	1.40	1.64	5.19
3 / iid	2.72	4.50	1.92	4.70

# Notes on comparison

- BART didn't assume anything about the Xs
  - In design 1, the other methods used only main effects
  - In design 2 & 3 they used all 2-way interactions, but the dimensionality ( $p$ ) was small (4 and 6)
- Can you use such models with 50 predictors? That's over 1200 2fi's to consider.

# Closing thoughts

- Does zero noise change anything?
  - Maybe not if we view importance measures as summaries, not inferential procedures.
- Does form of the model matter?
  - Maybe not, if everything is done in terms of the training points, and the model is flexible enough to interpolate.
  - In which case, GPs may be an expensive solution for large datasets.
  - High dimensionality may make interactions hard to work with.
- Thanks for stimulating talks and papers!!